

Write a proof for each proposition. Use complete sentences.

1. **Proposition:** If $a \mid b$ and $c \mid d$, then $ac \mid bd$.

2. **Proposition:** Suppose A and B are sets. Then $A - (A - B) = A \cap B$.

3. **Proposition:** The number $\log_2(3)$ is irrational.

Hint: Use proof by contradiction and the fact that $\log_2(3) > 0$.

4. **Proposition:** There exists a set X such that $X \cap \mathcal{P}(X)$ is not empty.

Hint: What element is in $\mathcal{P}(X)$, no matter what X is?

5. **Proposition:** If $3 \nmid n$, then $3 \mid (n^2 - 1)$.

Hint: Divide into cases.

6. **Proposition:** For all integers $n \geq 1$, $\sum_{i=1}^n 3^i = \frac{3^{n+1} - 3}{2}$.

7. **Proposition:** For all integers $n \geq 1$, $\frac{d^n}{dx^n}(xe^x) = (x+n)e^x$.

8. **Proposition:** Suppose A and B are sets. If $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$, then either $A \subseteq B$ or $B \subseteq A$.

Hint: You may prove this directly, but proof by contrapositive may be easier.