

1. Consider the statement

Being a citizen and at least 35 years old are necessary to become president.

(a) Using natural language, rewrite the statement in the form “If  $P$ , then  $Q$ .”

(b) Write the contrapositive of the statement using natural language.

(c) Write the negation of the statement using natural language.

2. Consider the statement forms below.

$$(P \rightarrow Q) \rightarrow R$$

$$P \rightarrow (Q \rightarrow R)$$

(a) Use a truth table to determine whether or not the statement forms are logically equivalent.

| $P$ | $Q$ | $R$ |  |  |  |  |
|-----|-----|-----|--|--|--|--|
| T   | T   | T   |  |  |  |  |
| T   | T   | F   |  |  |  |  |
| T   | F   | T   |  |  |  |  |
| T   | F   | F   |  |  |  |  |
| F   | T   | T   |  |  |  |  |
| F   | T   | F   |  |  |  |  |
| F   | F   | T   |  |  |  |  |
| F   | F   | F   |  |  |  |  |

(b) Write a sentence explaining your answer.

3. Consider the sets

$$A = \{\triangle, \blacktriangle, \blacksquare, \blacksquare\}$$

$$B = \{\star, \blacklozenge, \blacktriangle\}$$

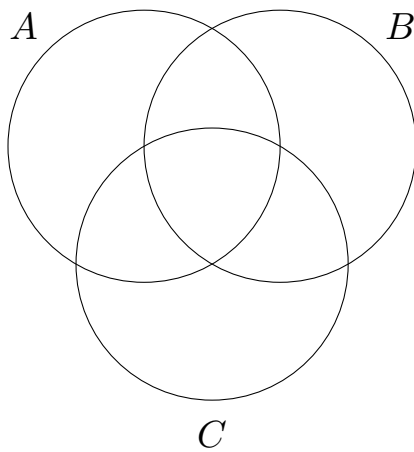
Determine whether each statement is true or false. Explain your answers with complete sentences.

(a)  $\forall a \in A, \exists b \in B, a$  is the same color or same shape as  $b$ .

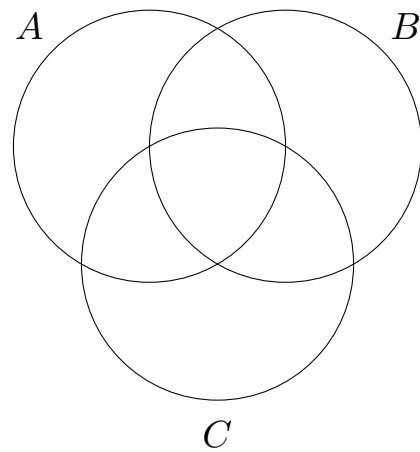
(b)  $\exists b \in B, \forall a \in A, a$  has fewer sides than  $b$ .

4. Shade the venn diagrams below to determine whether or not the sets are equal.

$$(A - B) - C$$



$$A - (B - C)$$



5. Consider the following definition for  $\lim_{x \rightarrow c} f(x) = L$ .

For any positive real number  $\varepsilon$ , there exists a positive real number  $\delta$  such that for all real numbers  $x$ ,  
if  $0 < |x - c| < \delta$ , then  $|f(x) - L| < \varepsilon$ .

Negate this definition. In other words, write what  $\lim_{x \rightarrow c} f(x) \neq L$  means.

6. Find the coefficient of the  $x^4$  term in  $(x - 2)^7$ . Simplify your answer.

7. Let  $A = \{x, y\}$ . List the elements of the following sets.

(a)  $A \times A$

(b)  $\mathcal{P}(A)$

(c)  $\mathcal{P}(A) \times A$

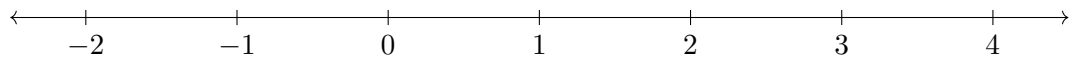
(d)  $A \times \emptyset$

(e)  $A \times \{\emptyset\}$

8. How many 5-permutations of  $\{1, 2, \dots, 9\}$  contain exactly two odd numbers? Explain your answer with complete sentences.

9. Let  $A_i = \left[-\frac{1}{i}, 3 - \frac{1}{i}\right]$ .

(a) Use the number line below to graph  $A_1$ ,  $A_2$ , and  $A_{100}$ .



(b) Find  $\bigcup_{i=1}^{\infty} A_i$ .

(c) Find  $\bigcap_{i=1}^{\infty} A_i$ .