

MATH 195: Gödel, Escher, and Bach (Spring 2001)

Problem Set 10: TNT and Quantifiers

To be discussed Thursday, March 22

10.1. Which of the following correctly pair a statement of number theory with a more elemental interpretation?

Statement of number theory	Elemental interpretation
a. 7 is not prime	There exists numbers c and d , both greater than 1, such that c times d equals 7.
b. A number is always less than its square	For every number b , that number b is less than b times b .
c. All multiples of 4 are also multiples of 2	For every number b such that there exists a number c where 4 times c equals b , there also exists a number d where 2 times d equals b .

10.2. Provide elemental interpretations of the following statements of number theory:

- 1363 is a multiple of 47
- No number is simultaneously even and odd
- There are an infinite number of even numbers
- Three times any number is always greater than two times any number

10.3. Interpret the following statements in simple English:

- There is no number **c** such that **c** times **c** equals 47
- There exists a number **a** such that there are no numbers **b** and **c**, both greater than 1, where **b** times **c** equals **a** and there are no numbers **d** and **e**, both greater than 1, where **d** times **e** equals **a** plus 2

10.4. Translate each statement below into English and determine whether or not it is true:

- $\exists c:(c+S0) = SS0$
- $\neg c:(c+S0) = SS0$
- $\exists a: \exists b: \exists c: (a = ((b \cdot b) \cdot b) + ((c \cdot c) \cdot c))$

10.5. Identify the free variable(s) and quantified variables in the examples of open and closed statements in English given below. Be sure to distinguish the open sentences from the closed sentences.

- x is an apple.
- x is an apple and $(\forall y)$ if y is a ripe banana, then y is yellow.
- $(\exists x)$ x is an apple and $(\forall y)$ if y is a ripe banana, then y is yellow.
- $(\exists x)$ y loves x .
- $(\forall x)(\forall y)$ if y loves x then x loves y .
- $(\forall x)(\exists y)$ x loves y and y loves x .

10.6. For each of the sentences from the last question, identify the property expressed (if the sentence is open) or the truth value (if the sentence is closed).

10.7. Translate the following sentences into a symbolic language as much like TNT as possible. You won't be able to translate predicates (leave them as English), but you should be able to translate logical relationships.

a. Computers are treacherous

b. Nobody makes a pizza like MamaMia!

c. Whosoever shall seek to save his life shall lose it; and whosoever shall lose his life shall preserve it.

d. Many are called, few are chosen (define "many" as ">10" and "few" as "<5")

e. You can fool some of the people all of the time, all of the people some of the time, but you can't fool all of the people all of the time.

10.8. Do the six translation puzzles on pp.212-213.

10.9. Does every formula of TNT require an equals sign?

10.10. Give three examples of atoms in TNT and two examples of non-atoms. What is it that makes the last two strings in the middle of page 207 be non-atomic?