

MATH 195: Gödel, Escher, and Bach (Spring 2001)

Problem Set 5: To be discussed Thursday, February 15

PS5.1. You are in possession of a map from Richmond to Chicago plus the desire to go to Chicago plus the opportunity to realize your desire. List the translations of meaning that must take place for you to use the map to get you to Chicago. Be sure to map information to information (not object to object).

PS5.2. Write out as many connections as you can between the two halves of *Contracrostipunctus*.

PS5.3. How would you translate Axiom Schema II into arithmetic, using the isomorphism noted in Chapter II?

PS5.4. Hofstadter claims that $\neg p \rightarrow \neg q$ is a theorem in the new system. Justify this claim using only formal rules.

PS5.5. I've recently become anxious about certain thoughts that you are most unlikely to have heard of. I call them "thoughts which, if thought by me, would leave me unable to reason correctly". I am especially worried about such thoughts when I am assigning partial credit to student-written proofs. My method, you see, is as follows:

1. Follow the student's proof until the first error.
2. Attempt to jump inside the student's head, continuing from the error in a logical fashion until the next error is reached.
3. Jump again, continuing from the second error in a logical fashion until the next error is reached, You should realize that once the process is started, I am in the student's power - I cannot stop reading until I reach the end of the proof.
4. Jump outside the student's error-filled attempt in order to assign partial credit based on the number and seriousness of the errors encountered.

Consider Figure 20 with its sort of Venn diagram on the right hand side. Draw a sketch in the manner of Figure 20 to illustrate the feedback loop involving "my thoughts upon reading the student's proof" and "thoughts detrimental to my reasoning ability". What would a devilish student have to know in order to hurt me?

PS5.6. Answer the question posed by Hofstadter at the bottom of p102. Give some justification for your response.

PS5.7. Consider the **pq**-system with the following additional axiom schema:

AXIOM SCHEMA II: If x is a hyphen-string, then $xp - qx$ is an axiom

- a. From the two axiom schema (the original and II) derive three new theorems
- b. Is this new system consistent with the external world?
- c. Can you think of any interpretation to make it consistent?

PS5.8. Draw a sketch similar to Figure 20 with goblet G taking the place of the phonograph + record. Describe the chain of isomorphisms (correspondences) that lead from the goblet back to the goblet as best you can.

PS5.9. Your friend from Maryland is late coming for dinner. He gives as an excuse that he got lost because he never saw any big blue line crossing I-95, so he thought he hadn't made it into Virginia yet. "I was following the map precisely," he says earnestly. "I don't know what could have gone wrong!"

- a. Explain to him, in terms of explicit and implicit meaning, what went wrong.
- b. OK, that was a fairly stupid example. Think of cases in real life where people confuse symbols or words for what they're meant to represent. There are many, and we get into a lot of trouble because of them.