MATH 195: Gödel, Escher, and Bach (Spring 2001)
Common problems on Homework #2

PS5.4. Hofstadter claims that **- - p - - q - - -** is a theorem in the new system. Justify this claim using only formal rules.

Where to start? Ask yourself:

- *Is the string a theorem in the OLD system? (no)*
- *Then, if it is a theorem in the NEW system, it must be so by virtue of the difference between the new and the old.*
- *What IS the difference between the new system and the old? That's your starting point.*

PS6.1. Doll on top of a doll on top of a doll…. The first doll is 1 ft. tall. Each successive doll is half the size of the previous. What's the height?

It isn't enough to give the answer. The PROCESS is much more important. How do you KNOW the final height. OK, so maybe each doll gets smaller and smaller and you think that the final height is finite. But why isn't the height 3.5 ft or any other number?

PS6.3. Apply the phone number lookup process to the faculty section of the University phone directory, using either Kerckhove or Elhai as the name to look up. Simplify your life by using some estimation of the midpoint name. Which name is found in the fewer number of steps? Among the faculty and staff, whose name(s) are the "worst" to look up (i.e. require the greatest number of steps)?

Many of you simplified the problem so that it almost disappeared. Whatever midpoint procedure you use, it has to be systematized, capable of being performed by a machine that is unable to recognize patterns (i.e. scan for a name). How do you know when to stop the recursive procedure? This may be obvious to you, a human, but you need to make the end point so explicit that a machine could understand. Only by defining the endpoint can you understand the answers to the questions regarding steps and worst name. Above all, don't talk theoretically about an answer. DO IT!

PS6.9. Provide a set of rules to be used with the symbols of the **MIU**-system and the sole axiom **MU** (<u>not</u> **MI**) that can generate as many strings as possible that are NOT theorems in the **MIU**-system (with **MI** as the sole axiom). Post at least one rule to the Discussion Board.

It is important not to lose sight of the overall goal: to generate nontheorems. The strategy is:

a. *Find rules that can take a nontheorem and make a new nontheorem*
b. *To start with **MU**, because we know that's a nontheorem (from Exam 1)*

*How can you devise rules that you are <u>certain</u> produces nontheorems from nontheorems? (You don't care what they do with theorems). You can't make them up indiscriminately. The only things you know within the **MIU-**system are the theoremhood of the axiom **MI** and the rules of generation. THEREFORE, the force of these new rules (perhaps they should be called tools) must stem from the preexisting axiom and the four rules of generation. Can you find a rule that must produce nontheorems from other nontheorems, <u>given (say) that **MxUUy → Mxy**</u>?*