MATH 195: Gödel, Escher, and Bach (Spring 2001)

Problem Set 13: Gödel numbering

To be discussed Tuesday, April 17

1. Decide whether each of the following statements is true or false. Provide an explanation and either a complete example or complete counterexample. ("complete" means all elements of any relevant isomorphism)

a. Every **TNT**-number is a number.

b. If *x* is a **TNT**-number, then *x* is the Gödel Number for some string of **TNT**.

c. If *x* is the Gödel Number for some particular string of **TNT**, then *x* is a **TNT**-number.

2. Find a number, other than numbers in the book,

a. that is a **TNT**-number.

b. that is presumably not a **TNT**-number.

3. Say which of the following numbers are **TNT**-numbers and what leads you to believe so.

a. 2121236

b. 666111210

c. 333262636362323262111262

d. 262111123666

4. Make up an **MIU**-string, convert it to its Gödel number, and make up a statement claiming that the number is the product of some specific arithmetized rule of the **MIU**-system. Write that statement in TNT language (shortcuts are allowed as necessary!).

5. Figure out an arithmetized version of the Double-Tilde Rule. (Hint: gain inspiration from the somewhat similar arithmetized Rule #4 of **MIU**, which is described in GEB).

6. Diagram the procedures that take the TNT string *H* to the TNT string *H'*, in the form of a Recursive Transition Network.

7. Go through the notes for April 12 and write in one column the steps used to take a string, ***x***, of **MIU** to a corresponding string of **TNT** that, when interpretted, says that **x** is a theorem of **MIU**.

8. (Connected to #7) Write in the second column the steps used to take a string, ***H,*** of **TNT** to a different string of **TNT** that, when interpreted, says that ***H*** is a theorem of **TNT**. Make sure that series of steps in this column line up with the corresponding series of steps produced in #7.

9. Compose a problem set-like question that gets at the core of Gödel numbering of formal systems and post it to the bulletin board. The question should be (perhaps barely) within reach of your colleagues, but you don't necessarily have to be able to answer it yourself.

10. Show how the following numbers satisfy the Goldbach property:

a. 32 b. 38 c. 98

11. Show how the following numbers satisfy the Tortoise property:

a. 32 b. 38 c. 98

12. Devise a strategy that will check any number for the Goldbach property. Write it as a flow chart or recursive transition network.