

MATH 198: Gödel, Escher, and Bach (Spring 2000)

Problem Set 1: *Logic Puzzles*

To be discussed Tuesday, January 23

1. Fill in the blanks so that this self-referential sentence is true. Your solution should be different from the one given in class.

In this sentence, the number of occurrences of 0 is 1, of 1 is __, of 2 is __, of 3 is __, of 4 is 1, of 5 is 1, of 6 is 1, of 7 is __, of 8 is 1, and of 9 is 1.

2. These sentences take a peculiar interest in each other's contents! Fill in the blanks to make each sentence true.

- *In the next sentence, the number of occurrences of 0 is 1, of 1 is __, of 2 is __, of 3 is 1, of 4 is 1, of 5 is 1, of 6 is 1, of 7 is 1, of 8 is __, and of 9 is 1.*

- *In the previous sentence, the number of occurrences of 0 is 1, of 1 is __, of 2 is __, of 3 is 1, of 4 is __, of 5 is 1, of 6 is 1, of 7 is __, of 8 is 1, and of 9 is 1.*

3. I offer two prizes --- Prize 1 and Prize 2. You are to make a statement. If the statement is true, then I will give you one of the prizes (I'm not saying which one). If your statement is false, then you get no prize. Obviously you can be sure of winning a prize by saying, e.g., "Two plus two is four," but suppose you have your heart set on Prize 1; what statement could you make that would guarantee that you will get Prize 1? (*Hint: Consider a statement of the form "You will not give me. . .", then reason if the statement is false then. . . while if the statement is true then....*)
4. Another game with two prizes. This time, if you make a true statement, you get Prize 2; if you make a false statement, you don't get Prize 2 (you may or may not get Prize 1). What statement will get you Prize 1? (*Hint: there are four possible outcomes to the game in terms of what you get, and the statement to make is similar in form to the one in problem 3*).
5. Make up a strange loop, either in language or some other medium. The stranger the better. Try posting your favorite answer on the Discussion Board.