

MATH 195: Gödel, Escher, and Bach (Spring 2000)

Notes and Study Questions for Thursday, March 8

Reading: Dialog - *Crab Canon* (pp.199-203)

Chapter VIII - *Typographical Number Theory* (pp.204-206; up to *Numerals*)

Homework (handing in optional): Problem Set 9: 1a, 2a, 3a, 4a, maybe 5

You are ALWAYS invited to hand in your work on the problem sets. However, since this one is not required, there won't be credit given for it on the exam, except the indirect credit that comes from knowing how to do the problems better.

Crab Canon

The dialogue is easy to read without much thought, but becomes more interesting when you pay more attention to it.

SQ1. Why is the dialogue called *Crab Canon*? Relate the structure of the dialogue to that of Bach's *Crab Canon* and/or Escher's *Crab Canon* (see Fig. 44 and listen to the file in Additional Material on the web).

SQ2. Are there places where the structural isomorphism is not exact? Why were those differences inserted?

Mysteriously interpolated into the dialogue is a brief explanation of the structure of DNA. Everyone is no doubt aware that DNA is a double-stranded molecule that contains all the genetic information within an organism. You might not realize that this information is contained twice -- once on each strand. Since the letters within DNA (G, A, T, C) pair with their counterparts (G with C, A with T), one strand is the inverse (or background) of the other. In the sequence shown in Fig. 43, one strand, read left to right, is identical to the other strand, read right to left. In the cell, the two strands are in fact read in opposite directions, and such crab canon-like structures are of immense importance in the function of DNA.

SQ3. Why do you think the structure breaks down midway (when Crab enters)?

SQ4. Why do you think Hofstadter has Crab say "TATA" at the end rather than "Goodbye"?

CHAPTER VIII: Typographical Number Theory

Typographical Number Theory is not just one of several formal systems introduced in the book -- it is THE formal system. All the others were leading up to this one. It is this formal system that Gödel is going to blow up with the Record That Cannot Be Played On Record Player TNT. In order to appreciate the event, however, we need to see why the Crab might view this system as the Perfect Record Player.

That business is for after Spring Break. Right now I want us just to touch on the basic mechanics of the system so that you might feel more comfortable reading the chapter.

SQ5. The interpreted **PQ**-system makes statements about integer addition, the interpreted **TQ**-system makes statements about integer multiplication, and who knows what the **MIU**-system says. What does the interpreted **TNT**-system talk about?

The objectives of the **TNT**-system are far grander than any previous we've encountered. It is reasonable, then, that its symbols and rules are more complex. Today I'm concerned only with the symbols. The atoms of the system should strike you as similar to those of the **PQ**-system. In the latter system, numbers were represented as a series of hyphens. In the **TNT**-system, numbers are represented as a series of **S**'s preceding **0**.

SQ6. Why was the letter **S** chosen?

SQ7. What numbers can be represented in the **TNT**-system? Do they differ from the numbers that can be represented in the **PQ**-system?

That's it! Spend the rest of your time on Problem Set 9!