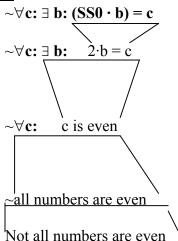
## MATH 195: Gödel, Escher, and Bach (Spring 2001)

Typographical Number Theory: Translations and Well-Formed Formula

## Hints for translating TNT sentences to English

- Don't try to understand an entire sentence all at once
- Understand chunks of a sentence; replace chunks as you go along
- Usually it's easier to go right to left
- Test reasonableness as you go along

## **Example**



Two times  $\mathbf{b} = \mathbf{c}$ ... reasonable, but not so interesting. There EXISTS some  $\mathbf{b}$  for which this is true... If so, then  $\mathbf{c}$  must be even

For every c, c is even...doesn't sound reasonable to me, but that's what it says. If it's true for every number c, then it's true for all numbers.

...which is a true statement. Note that negating the statement, "All numbers are even" does NOT give "No numbers are even."

## **Well-Formed Formula in TNT**

Do the symbol strings below qualify as "well-formed formulas of TNT"?

1) 
$$\exists b: \langle a = SS0 \land (a + b) = S0 \rangle$$
 compare with  $\langle a = SS0 \land \exists b: (a + b) = S0 \rangle$ 

- 2)  $\forall \mathbf{a} : < \mathbf{a} = \mathbf{SS0} \supset \exists \mathbf{b} >$
- 3)  $(\exists b: \exists c: SSb \cdot SSc = \sim a)$  compare with  $(\sim \exists b: \exists c: SSb \cdot SSc = a)$
- 4)  $< S0 = 0 \lor S0 >$