MATH 195: Gödel, Escher, and Bach (Spring 2001)

Typographical Number Theory: Translations and Well-Formed Formula

Hints for translating TNT sentences to English

- Don't try to understand an entire sentence all at once
- Understand chunks of a sentence; replace chunks as you go along
- Usually it's easier to go right to left
- Test reasonableness as you go along

<u>Example</u>

$\sim\forall \mathbf{c}\mathbf{:}\ \exists\ \mathbf{b: (SS0 \cdot b) = c}$

$\sim\forall \mathbf{c}\mathbf{:}\ \exists\ \mathbf{b:}\quad 2 \cdot b = c$

*Two times **b** = **c**… reasonable, but not so interesting.*
*There EXISTS some **b** for which this is true…*
*If so, then **c** must be even*

$\sim\forall \mathbf{c}\mathbf{:}\quad$ c is even

*For every **c, c is even…doesn't sound reasonable to me, but that's what it says.** If it's true for every number c, then it's true for all numbers.*

~all numbers are even

Not all numbers are even

…which is a true statement. Note that negating the statement, "All numbers are even" does NOT give "No numbers are even."

Well-Formed Formula in TNT

Do the symbol strings below qualify as "well-formed formulas of **TNT**"?

1) $\exists \mathbf{b: < a = SS0 \wedge (a + b) = S0 >}$ *compare with* $\mathbf{< a = SS0 \wedge \exists b: (a + b) = S0 >}$

2) $\forall \mathbf{a:< a = SS0 \supset \exists b >}$

3) $\mathbf{(\exists b: \exists c: SSb \cdot SSc = \sim a)}$ *compare with* $\mathbf{(\sim \exists b: \exists c: SSb \cdot SSc = a)}$

4) $\mathbf{< S0 = 0 \vee S0 >}$