

Rules of Derivation for TNT

All the rules for the Propositional Calculus are also rules of TNT

Rule of Specification:

If $\forall u: x\{u\}$ is a theorem, then so is $x\{u\}$ and so is $x\{v/u\}$ where v is any term

Restriction: v cannot be quantified within $x\{u\}$

If it's true for all, it's true for one.

Rule of Generalization:

If $x\{u\}$ is a theorem, then so is $\forall u: x\{u\}$

Restriction: u must be free

If something is true for something without restrictions, then it's true for all.

Rule of Interchange:

The strings $\forall u: \sim$ and $\sim \exists u:$ are interchangeable

If it isn't true for all, then there doesn't exist any for which it's true

Rule of Existence:

Any instance of a term may be replaced by u , if $\exists u:$ is placed in front.

Restriction: The term cannot contain variables that aren't free.

Rules of Equality:

Symmetry: If $r = s$ is a theorem, then so is $s = r$

Transitivity: If $r = s$ and $s = t$ are theorems, then so is $r = t$

Rule of Successorship:

Add S: If $r = t$ is a theorem, then so is $Sr = St$

Drop S: If $Sr = St$ is a theorem, then so is $r = t$

Rule of Induction:

If $\forall u: \langle x\{u\} \supset x\{Su/u\} \rangle$ is a theorem

and $\langle x\{0/u\} \rangle$ is a theorem

then $\forall u: x\{u\}$ is a theorem

Restrictions: $x\{u\}$ is well-formed; u is free