MATH 195: Gödel, Escher, and Bach (Spring 2000)

Notes and Study Questions: Recursive Structures and Processes
Reading: *pp.127-136*, *146-152* (to be discussed Tuesday, February 20)

Notes

Here is a chapter that will thoroughly confuse you *unless you follow along by doing the examples*. Recursive transition networks are easy. After all, even dumb computers can do them. You can too, so long as you don't attempt to understand everything at once. Go step by step through the procedures.

We will be collecting on Tuesday your responses to the problems listed below. As before, you'll get up to 10 points on the upcoming exam for your effort. If you turn it by Thursday, you'll get full credit, but then you probably won't get back our feedback before the next exam.

Problem Set 5: #4, #8
Problem Set 6: #1, #3, #9

Notice that the last problem asks you to post something to the Discussion Board.

What is recursion? Pushing, popping, and stacks

SQ1. Some of the following are examples of properly formulated recursive definitions and others are examples of circular definitions. Which are which?

a. A recursive definition is one that is recursive

b. To make a series of concentric circles, draw a circle with a radius of one unit, then draw other circles with the same center and a radius one unit greater than that of the previous circle

c. Here's a (rather poor) way to understand a sentence: (a) Read the first unread word in a sentence, (b) understand its meaning, (c) go back to step (a).

SQ2. Why is it important for a recursive definition to define a term / concept in terms of *a simpler version* of itself?

SQ3. On p.127, Hofstadter characterizes his description of the executive's behavior as recursion in its most precise form. What would be a less precise form? What problems would such behavior raise in a purely mechanical executive?

SQ4. Over lunch or dinner, discuss cafeteria trays and push-down stacks with the people at your table.

SQ5. On p.128, the news broadcast switched from news central to Sally Swumpley in Peafog and then to Nigel Cadwallader just outside of Peafog. Contrast the pushing and popping in the executive telephone example to the pushing and popping that typically takes place in news broadcasts.

Stacks in music; Recursion in language

SQ6. Consider *Three Blind Mice*. Diagram the song, using the line numbers below and Figure 26 as a model.[1]

1. *Three blind mice, Three blind mice*
2. *See how they run, See how they run*
3. *They all ran after the farmer's wife*
4. *Who cut off their tails with a carving knife*
5. *Did you ever see such a sight in your life*
6. *As three blind mice*

Hofstadter did not give any examples of the German complexities he alludes to. Here is a simple example of a German sentence with a verb at the end:

Ich habe vor drei Jahren in Bonn gewohnt.

I have for three years in Bonn lived.

You get the idea that German-English is Yoda-speak. Here is a more complex example (English literal translation only):

*The trunks being now ready, he **DE-** after kissing his mother and sisters, and once more pressing to his bosom his adored Gretchen, who, dressed in simple white muslin, with a single tuberose in the ample folds of her rich brown hair, had tottered feebly down the stairs, still pale from the terror and excitement of the past evening, but longing to lay her poor aching head yet once again upon the breast of him whom she loved more dearly than life itself, **PARTED**."*[2]

The relevance to the chapter, however, are the stacks of clauses that may arise in complex German sentences, each popping out with a verb at the end. I don't know German, so I've offered an English sentence followed by my conception of how it might be rearranged in German as stacks of clauses:

English: *German sentences that have subordinate clauses, which may be long or short, must put several verbs at the end.*

German-English(?): *Several verbs at the end must German sentences (that subordinate clauses(, which may long or short be) have) put.*

SQ7. Find someone who really knows German with whom to discuss the "verb-at-the-end" phenomenon. Construct an English sentence that illustrates this phenomenon. Construct a ridiculously long English sentence that home the point really well drives.

---

[1] Note to Saadia: If *Three Blind Mice* isn't in your mental collection, try the same thing with a children's song you DO know. I hope your songs are more kindhearted than ours.

[2] This example was taken from Mark Twain's, *The Awful German Language*. You can read the whole thing if you like by clicking on a link in Additional Material.

Recursive Transition Networks; "Bottoming out"; Expanding nodes

SQ8. Step through the flow charts of Figure 27, producing several examples of ornate and fancy nouns. Resist the temptation to post your creations on the discussion board for the class.

SQ9. Revisit SQ1 in light of Figure 27: What is the crucial fact that distinguishes recursive definitions from circular ones?

SQ10. Draw a picture that illustrates the concept of "expanding a node" in the context of Meta-Genies and Djinn.

Diagram G and Recursive Sequences (only pp.135 and 136)

SQ11. Do you see how you get from Figure 29a (Diagram **G**, unexpanded) to Figure 29b (Diagram **G**, expanded once)? If not, write out three copies of Figure 29a -- call them the first "unexpanded" and the other two "copies". Put one of the copies on a "**G**" of the unexpanded diagram, putting the lower black dot on top of the "G". Do the same with the other "**G**" of the unexpanded diagram, putting the second copy on top of it. You should now have Figure 29b.

SQ12. Write out the numbers 1 through 13 and below them the first 13 Fibonacci numbers. For which numbers, n, does **FIBO**(n) = n?

SQ13. What is the 14th Fibonacci number?

SQ14. Use the recursive RTN of Fig. 31 to calculate the 5th Fibonacci number (forget that you already know what it is). Write out and keep track of each copy of x and y on the stack.

Copies and Sameness; Programming and Recursion

SQ15. Hofstadter discusses copies and sameness --- does his notion of "sameness" agree with our previous conception of "kind-of-the-same"? In what ways are the two the same? In what ways are they different? Illustrate your responses using fish or butterflies if you like.

SQ16. What is a *loop* in the sense of computer programming? What is the difference between a bounded loop and a free loop? Is the "loopiness" in ORNATE NOUN bounded or free? What about FANCY NOUN?

SQ17. Why does Hofstadter prefer to describe ORNATE NOUN as a procedure rather than a loop?