MATH 195: Gödel, Escher, and Bach (Spring 2001)
Notes and Study Questions for Tuesday, February 13

Reading: *Contracrostipunctus* (pp.75-81)
Chapter IV: *Consistenty*, *Completeness*, *and Geometry* (pp.82-88)

Notes

I find this reading to be the most difficult to date because, paradoxically, it is the easiest. It concerns itself with verbal analogies, and its so tempting to say, "Yeah, that's sort of right" and move on. The hard part is to stop yourself and say, "Wait a second. . . what exactly does he mean by that?"

You can start with *Contracrostipunctus*. It's easy to read it as a moderately pleasant conversation between a reptile and a long dead warrior, but if you're awake, you'll readily see that there are strange interconnections between different parts of the story and a lot that could be interpreted symbolically. As you go along, make note of any internal isomorphisms you encounter (e.g., the tortoise in one part of the story is isomorphic in some way to the crab in another part).

Contracrostipunctus

This story gets its name by combining two words, both related to Bach:

- *Contrapunctus*, the last piece Bach wrote; part of *The Art of the Fugue*
- *Acrostic*, a puzzle in which a message is concealed in the first letter of words. Bach reportedly wrote such puzzles.

Both meanings are important in the story.

SQ1. I suppose that the phonograph is going/has gone the way of the telegraph and the spinning wheel. . . you do know what one is, don't you?

SQ2. Can you think of a way the tortoise could outwit Crab's Record Player Omega?

SQ3. At some point, the story seems to shift gears completely. Where is that?

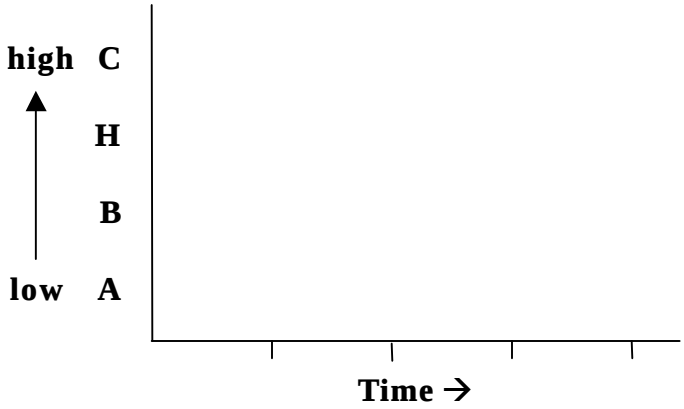
SQ4. On p.79 the story brings in an isomorphism between letters and musical notes and an isomorphism between the German system of musical nomenclature and that used by the rest of the world. What are the two isomorphisms?

We think of the musical note **B** as having two forms: **B** (or **B**♮, **B**-natural) and the lower **B**♭ (**B**-flat). In Germany, the natural form is called **H**, while what we call **B**♭ is given the name **B** (notice the similarity on one hand between a hard lower case **h** and the symbol ♮ and on the other between a soft lower case **b** and the symbol ♭). So, in Germany, the order of notes is **A B H C D E F G**.

I've put *Contrapunctus* in Additional Material, along with the **B-A-C-H** theme. Even if you can read music and follow along, it's not at all easy to pick out **B-A-C-H** at the end!

SQ5. Plot the melody **B A C H** on the graph at the right, connecting the dots you draw.

SQ6. Draw above the figure you just drew the shape of the same figure, only upside down. Draw the same shape only backwards. Which is the same as the original?



SQ7. The goblet bursts into pieces. . . any connection with a previous part of the story? Use this event to draw as many cross connections as you can.

SQ8. In the unlikely event that a dialogician should write a contrapuntal acrostic in homage to J.S. Bach, do you suppose it would be more proper of him to acrostically embed his OWN name – or that of Bach? Do you see how Hofstadter's answer to this question "boomerangs"? Is the acrostic a true statement?

Implicit and Explicit Meaning

SQ9. "In Chapter II, we saw. . . The more complex the isomorphism. . . the more `equipment'. . . is required. . ." Syntax is right. Sounds good. But what does this mean? Make specific connections to examples from previous chapters.

SQ10. "This `explicit' meaning is, strictly speaking, extremely implicit. . ." Explicit. . . implicit. . . what does it all mean? What is the most explicit meaning of the story?

This section is a bit misleading in my opinion, speaking of two levels of meaning. You might get the idea that one level is more basic than the other, but this isn't what's meant. Perhaps it would be better to refer to two parallel pathways of meaning, one pathway leading to connections in your brain and the other leading to destruction of a phonograph.

SQ11. Diagram all the translations of meaning that take place in between the physical phonograph record and your perception of music.

SQ12. Now with Hofstadter's help, revisit SQ7 and write out as many connections as you can between the two halves of the story.

SQ13. List other ways in which the story is self-referential.

Mapping between *Contracrostipunctus* and Gödel's theorem

SQ14. What parts don't you understand about the connections with logical systems drawn in the middle of p.85?

The modified pq-system and inconsistency

SQ15. How would you translate Axiom Schema II into arithmetic, using the isomorphism noted in Chapter II?

SQ16. Hofstadter claims that **- - p - - q - - -** is a theorem in the new system. Justify this claim using only formal rules.

SQ17. In the new system, **- p - q - -** is a theorem and so is **- p - q -**. Hofstadter points this out and concludes that the new system is internally inconsistent. Explain.

SQ18. At the bottom of p.87, Hofstadter claims that all the problems with the new system can be made to vanish with a suitable reinterpretation of the system. Suppose that you just don't see any connection with the real world that works with the system. Is that system therefore logically inconsistent?