MATH 195: Gödel, Escher, and Bach (Spring 2001)

Notes and Study Questions for Tuesday, February 4

Reading: *Sonata for Unaccompanied Achilles;* Chapter III – *Figure and Ground*. We'll also discuss isomorphisms and meaning (pp.49-54)

Music (optional): Bach's *Sonata for Unaccompanied* Violin (in Additional Material)

We've been worrying about how to derive theorems from axioms. Except in the case of **MU** itself, we've not spent time worrying about how to prove that a string is NOT a theorem (and proving it for **MU** was hard!). But isn't it always possible to say: If I find a way to describe the set of all theorems, then nontheorems are just everything else! The set of nontheorems would seem to be the negative image of the set of theorems. In particular the set of all prime numbers would seem to be the negative image of the set of nonprimes. Are these two sets the same sort of beast? Chapter III says maybe not.

By the way, you're not going crazy, Study Questions 1 through 6 below ARE the same as ones you saw last week.

Isomorphisms Induce Meaning; Meaningless and Meaningful Interpretations

1. Is the pq-system equivalent to addition? Why or why not?
2. Maps are examples of documents in which symbols are given meaning through an isomorphism. Give examples of symbols used in maps and their interpretations. Note that this "language" isn't very complex: it would be unusual to make a sentence out of symbols from a map (but give it a try!). Think of other instances where symbols are given meaning via an isomorphism.
3. How do we know which interpretation to apply? What's wrong with p ←→ horse?

Active *vs.* Passive Meanings and Double-Entendre

4. Try out the alternative interpretation of the symbols of the pq-System suggested at the bottom of p.52. How can you be sure that all the old theorems are still true?
5. What is the difference between meaning in a formal system and meaning in a human language?
6. What is the difference between active and passive meaning?

Sonata for Unaccompanied Achilles

(OPTIONAL): Write the Tortoise's part for this conversation. Send it to JElhai@Richmond.Edu, preferably well before Tuesday's class.

7. What was the tortoise doing that caused Achilles to twist his neck?
8. What was the tortoise's ingenious but degenerate solution to the "HE" puzzle?
9. What was the tortoise's second solution to the "HE" puzzle?

10. What is the solution to the "ADAC" puzzle?

Primes vs Composites; The tq-system

11. Don't do it, but would you have trouble listing which of the first 30 positive integers are prime numbers and which are composite numbers?
12. Define the **tq**-system, by naming the (a) legal symbols, (b) axiom(s), and (c) rule of production, all in symbolic form.
13. Hofstadter claims that the **tq**-system is supposed to capture multiplication. Does it? Translate the axioms to arithmetic statements. Translate the three theorems shown on p.65 into arithmetic statements.
14. Why is the rule of inference from of the **tq**-system arithmetically reasonable?

Capturing Compositeness and Illegally Characterizing Primes

15. Hofstadter implicitly introduces a new symbol into the **tq**-system in order to state a new RULE. What is the symbol, and what is the complete set of allowed symbols for what the new **C**-system? Are you of the opinion that variables ought to be included in the list?
16. What is the arithmetic meaning of the new rule, and how does it capture compositeness? Why does the new rule have the two hyphens in it? Wouldn't ***x*** **t** ***y*** **q** ***z*** work just as well to express compositeness? (Think of a special case)

Figure and Ground

17. Comment on the Smoke Signal drawing, p.702, as it relates to what we've done so far. Try to see a message that says "*Ceci n'est pas un message*" (meaning "This is not a message"). You might also compare this figure with the one on p.494.
18. Consider the definitions of "cursively drawable" (by the way, what would be "cursively undrawable" or "uncursively drawable"?) and "recursive" on p.67. Draw a Venn diagram that shows both the relationship between the two and the meaning of the statement on p.68 "*There exist cursively drawable figures that are not recursive*".
19. What puzzle does Fig. 17 solve? Do you see that it solves it?
20. *Explain Figure 18 as you understand it to your roommate, playmate, soul mate, shipmate, or even a random passerby. This is very important!*