

## **MATH 198: Gödel, Escher, and Bach (Spring 2000)**

### Notes and Study Questions for Thursday, January 18

Reading: *Intro - A Musico-Logical Offering* (pp.3-28), but we will not cover in this course the material introduced by the section entitled *Babbage, Computers, Artificial Intelligence* (pp.24-27).

There are multiple purposes of these notes and study questions:

1. To encourage you to read actively, eventually gaining the habit of generating your own study questions;
2. To encourage you to read relevant sections and consider them deeply BEFORE coming to the class in which they will be discussed (by you);
3. To direct your attention to points worthy of your consideration (though not implying that other points are not so worthy);
4. To help focus the discussion in class;
5. To give us an idea of what concepts are most in need of discussion.

This book places a great emphasis on integration of concepts at different levels. Often it is not possible to recognize connections until long after they're first presented. I advise you to keep a notebook page devoted to Open Mysteries that you check periodically to see if you are able to solve any of them. Not all the study questions are readily answerable. You may want to save the ones you can't answer also as Open Mysteries.

#### **Study Questions** (Be prepared to discuss them Jan 18)

1. What property does "*Row, Row, Row Your Boat*" and "*Frere Jacques*" have in common? Would any tune work just as well? Suggest one as a test case.
2. Hofstadter claims that *Good King Wenceslas* works in a canon as an inversion. You can decide for yourself. Click on the song in *Course Documents, Additional Material*.
3. What's the connection between Bach's endlessly rising canon and Escher's *Waterfall*?
4. A quote from GEB (p15): "*Implicit in the concept of Strange Loops is the concept of infinity, since what else is a loop but a way of representing an endless process in a finite way?*" There's a lot to think about here. Try to elaborate on your conception of infinity, either musically, artistically, or through language. Consider using Escher's *Metamorphosis, Mobius Strip I*, or *Mobius Strip II* (p276) to assist you. For additional help, you might recall the *Autumn Floods* section of Chuang-Tzu: "*The Lord of the River said, 'Men who debate such matters these days all claim that the minutest thing has no form and the largest thing cannot be encompassed. Is this a true statement?'*" Are there things about infinity that you find paradoxical? It may be two class periods before we consider infinity in conjunction with Zeno's paradox, but it will be good to get started on this big topic.

5. Are the following statements true or false? What's the problem?  
*"All cretans are liars."* -- uttered by Epimenides the Cretan  
*I always lie.*  
*This statement is false.*
6. Translate the following sentence into language the guy on the street could readily understand:  
*All consistent axiomatic formulations of number theory include undecidable propositions*
7. Accept for the moment Hofstadter's statement that "This statement of number theory does not have any proof in the system of *Principia Mathematica*" can indeed be translated into a statement of number theory. We are interested in two questions:  
 1) Is it a true statement? And 2) Can it be proved in the system of *Principia Mathematica*? Describe the possibilities.
8. Consider **R**, the set of all run-of-the-mill sets (as defined in the text). Is **R** itself a run-of-the-mill set or is it a self-swallowing set?
9. Which Box Contains the Gold? Two boxes are labeled "A" and "B". A sign on box A says "The sign on box B is true and the gold is in box A". A sign on box B says "The sign on box A is false and the gold is in box A". Assuming there is gold in one of the boxes, which box contains the gold?

### **Vocabulary**

10. heterologocial: Is "*heterological*" (as defined in the text) heterological?
11. meta. . .: Compose a metasentence.
12. strange loop: Think of an example of a strange loop, besides what you've read. How tight is it?
13. consistent system: What would be necessary to make a system inconsistent?
14. complete system: What would be necessary to make a system incomplete?
15. infinity: What is its essential property?