

Welcomes you to Exam #4

Elements of Exam #4 System (Isomorphic to rules of previous exams)

UNIVERSALITY OF HUMAN THOUGHT PROCESSES AXIOM:

$$\forall P: \forall P': \forall Q: \langle\langle P' \supset Q \rangle \supset \langle P \supset Q \rangle\rangle$$

Any person who expends sufficient mental effort is capable of answering any question answerable by the person who posed it.

P: a person expends sufficient mental effort

P': a person who posed the question expends sufficient mental effort

Q: that person can answer the given question

INTERCHANGE EXCLUSION AXIOM:

$$\forall q: \forall p: \forall p'': \sim \exists p': \langle\langle \sim p=p' \rangle \wedge \langle \sim p'=p'' \rangle \rangle \supset \text{INTERCHANGE-TEST}\{p,p',q\}$$

For any question on the exam and all people taking the exam, there is no pair of nonequivalent people for whom the interchange test is true, unless at least one of them is an instructor for the course.

p: a person taking the exam

p': a person

p'': an instructor for the course

q: a question on the exam

INTERCHANGE-TEST{person #1, person #2, exam question}:

A BlooP test that determines whether person #1 and person #2 have exchanged information regarding the given question.

RULES OF WELL-FORMEDNESS

Rule #1: Any statement that appears printed or written in the book, notes, or anywhere else may also appear in the answer to any question.

Rule #2: Any statement that has the form of an answer to a question may appear on a Answer Sheet.

Rule #3: Any statement that does not have the form of an answer to a question may not appear on the Answer Sheet but may appear on a Thought Sheet.

Rule #4: Any statement that has the form of a question is yours to keep.

The Questions

1. (1) If you have neither received nor given aid regarding this exam, nor have you gained or given knowledge concerning a previous or future administration of this exam, then sign your name. Otherwise sign someone else's name.

2. (10) Have you completed and turned in the printed Evaluation of Instruction (to Nicole or equivalent)? Have you completed and submitted the online Exit Questionnaire?
3. (10) How many of the three last homework assignments (Problem Sets 12, 13, and 14) have you turned in?
4. (4) To how many of the last five quizzes have you submitted your responses?
5. (2) The book *Gödel, Escher, and Bach* by Douglas Hofstadter could reasonably have the subtitle:
 - A. *Limitations on the ability of formal logical systems to capture truth*
 - B. *Limitations on the ability of truth to capture formal logical systems*
 - C. *Limitations on the ability of fleet-footed warriors to capture plodding reptiles*
 - D. *Is truth lodged in the eye of the beholder?*
6. (8) Here is a string of Number Theory; a string that “talks about” a string of the **MIU**-system.

$$\langle 31 \text{ is an MIU-number} \supset \langle \exists m:\exists n: 311 = 3 \cdot 10^m + n \wedge n \text{ is less than } 10^m \rangle \rangle$$
 - a. What string of the **MIU**-system (symbols **M**, **I**, **U**) is being discussed?
 - b. What is the given string of Number Theory “saying”?
7. (8) TNT has an unexpected feature. Consider the two TNT statements below:

$$(1) \forall a:\sim Sa=0 \qquad \text{and} \qquad (2) \sim Sa=0$$
 - a. Which one of the following is true:
 - A. Statements (1) and (2) are both true
 - B. Statements (1) and (2) are both theorems, but only statement (1) is true
 - C. Only statement (1) is a theorem, and only statement (2) is true
 - D. Statements (1) and (2) are mutually contradictory
 - b. Which of the following conclusions follows solely from your answer to Question 7a? Draw a diagram to illustrate that conclusion.
 - A. TNT is incomplete with respect to number theory (reality)
 - B. TNT is inconsistent with respect to number theory (reality)
 - C. Number theory (reality) is incomplete with respect to TNT
 - D. Number theory (reality) is inconsistent with respect to TNT
8. (6) Provide a number of precisely 21 digits that is a TNT number.
9. (6) Provide a number of precisely 24 digits that is not a TNT number.
10. (12) Provide an arithmetized version of the DROP **S** rule.

11. (12) Consider the TNT statement:

$$\forall \mathbf{a}: (\mathbf{a} + \mathbf{SS0}) = \mathbf{S}(\mathbf{a} + \mathbf{S0})$$

- Translate this statement into English.
- Explain why the English statement is true.
- Provide a derivation for the TNT statement
- Describe what a TNT-proof-pair regarding this TNT statement would look like

12. (12) Consider a natural number, n , which may or may not be a power of ten.

- Diagram (as either a Recursive Transition Network or a flow diagram) a procedure that would test, using operations permissible in TNT, whether the number is or is not a power of ten. You may also use, if you like, the following simple program called COUNT-DIGITS(\mathbf{a}) which takes a number as input and outputs how many digits the number has. For example:

$$\text{COUNT-DIGITS}(1729) \rightarrow 4$$

- How many numbers would you need to test before you could say with confidence that 4747 is or is not a power of two TEN? Explain.
- Call that procedure POWER-OF-TEN-TEST. Is the procedure representable in TNT? Briefly explain your answer.

13. (6) Complete the phrase “When quined, . . .” in such a way that quining it produces a true statement. Write out both the phrase and the statement.

14. (8) Say whether each pair of numbers below, constitutes a TNT-PROOF-PAIR, and briefly explain why or why not. In each case, m represents the first number of the proof pair and n represents the second number. The spaces between numerals are there to improve readability (not to imply the end of one number and beginning of another).

a. $m = 666\ 111\ 666$
 $n = 666\ 111\ 666$

b. $m = 626\ 262\ 636\ 223\ 262\ 111\ 666\ 611\ 223\ 123\ 666\ 111\ 666$
 $n = 223\ 123\ 666\ 111\ 666$

15. (8) Translate the following statement of meta-TNT into TNT:

$$\sim \mathbf{S0=0} \text{ is a theorem of TNT}$$

You may use the BlooP program TNT-PROOF-PAIR{ \mathbf{a}, \mathbf{a}' } as appropriate.

16. (8) Arithmoquine (lower-case) the following TNT string: $\exists \mathbf{a}: \mathbf{Sa} = \mathbf{a}'$ (the answer should be a TNT string)

17. (12) The TNT string **G** (p.447), translated into number theory, can be represented:

(1) $\exists \mathbf{a}: \exists \mathbf{a}': \exists \mathbf{a}'': \dots$ (some massive system of equations involving **a**, **a'**, **a''**...)

where **a**, **a'**, **a''**, . . . represent all the variables in the long list of arithmetized rules required to derive **G**. We may presume that either the numbers, **a**, **a'**, **a''**, . . . exist to make the massive equation work out (making the statement of number theory true) or they do not exist (making the statement of number theory false).¹

Consider each of the cases given below, in each case stating what is implied regarding the completeness or consistency of TNT with respect to number theory.

- a. Suppose that **G** is a theorem of TNT and statement (1) happens to be false.
- b. Suppose that **G** is a theorem of TNT and statement (1) happens to be true.
- c. Suppose that **G** is not a theorem of TNT and statement (1) happens to be true.
- d. Suppose that **G** is not a theorem of TNT and statement (1) happens to be false. *This last one is rather more subtle than the others.*

¹If they both existed and did not exist, then number theory would be self-inconsistent, and if they neither existed nor did not exist, then number theory would admit that it is possible for neither **P** nor $\sim\mathbf{P}$ to be true -- both possibilities too horrible to contemplate.