VCU Discrete Mathematics Seminar

The Double-Critical Graph Conjecture

Dr Martin Rolek College of William & Mary

Tuesday, Mar. 27 12:30-1:20 (SPECIAL DAY & TIME) 4145 Harris Hall



A connected graph G is double critical if G is t-chromatic, but G - u - v is (t - 2)-colorable for every edge uv of G. A long-standing conjecture of Erdős and Lovász asserts that the complete graph on t vertices is the only double-critical, t-chromatic graph for all t. This conjecture is true for $t \leq 5$, and open for all other values of t. We will discuss the history of this conjecture and prove some of the classical results (the proof for t = 5 is already 30 years old!). More recent results have revealed numerous structural properties that non-complete double-critical graphs must satisfy. Using these properties, we will prove that the only claw-free, double-critical, t-chromatic graph is the complete graph for t ≤ 8 . Finally, we will look at a variation of Hadwiger's conjecture and show that any double-critical, t-chromatic graph contains the complete graph on t vertices as a minor for t ≤ 9 .

For the DM seminar schedule, see: http://www.people.vcu.edu/~dcranston/DM-seminar