VCU Discrete Mathematics Seminar

Finite subgraphs of uncountable graphs

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VCU!

Wednesday, Oct. 2
1:00-1:50
4145 Harris Hall

A major line of inquiry in infinite graph theory has been around the extent to which one can determine the global structure of an infinite graph simply by looking at its finite subgraphs. An early result in this direction, which is a consequence of the De Bruijn-Erdős compactness theorem, shows that if G is a graph with infinite chromatic number, then G contains finite subgraphs of every possible finite chromatic number. One can then define a function $f_G : \mathbb{N} \to \mathbb{N}$ by letting $f_G(k)$ be the least number of vertices in a subgraph of G with chromatic number k. f_G is an increasing function, and a natural question to ask is how quickly f_G can grow. Results of Erdős show that, for every function $f : \mathbb{N} \to \mathbb{N}$, there is a graph G with *countably* infinite chromatic number such that f_G grows faster than f. In 1982, Erdős, Hajnal, and Szemerédi asked if the analogous statement is true if we moreover require that G have *uncountable* chromatic number. We will answer this question and more broadly discuss other problems involving finite subgraphs of uncountable graphs.

For the DM seminar schedule, see:

`http://www.people.vcu.edu/~dcranston/DM-seminar/`