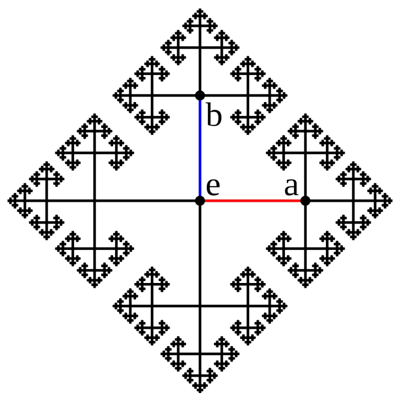
VCU Discrete Mathematics Seminar

Uncountable Triangle-free Graphs

Dr. Chris Lambie-Hanson, VCU

Wednesday, Dec. 5
1:00-1:50
4145 Harris Hall



When one first encounters graph coloring, it quickly becomes evident that the existence of many triangles (or, more generally, small cycles) in a graph poses an obstacle to the graph having a small chromatic number. One might then ask the question as to whether they pose the only obstacle: can a graph with no triangles (or no cycles of length less than 5, etc.) have arbitrarily large chromatic number? Of course, this question was answered in the relatively early days of graph theory, and classical graph constructions yield finite graphs of arbitrarily large girth and chromatic number. By taking disjoint unions of such graphs, we can push these results into the world of the countably infinite, producing countably infinite graphs of arbitrarily large girth whose chromatic numbers are infinite.

In this talk, we will consider some questions regarding generalizations of these results to the realm of the uncountably infinite. We will begin by noting a somewhat surprising way in which the results about finite and countable graphs cannot generalize to the uncountable case. We will then view some selections from a menagerie of uncountable graph curiosities, including a graph which is cycle-free and yet, in some models of set theory (without the axiom of choice), has uncountable chromatic number. We will end by outlining a construction which involves arguments both combinatorial and set-theoretic in nature and that yields, for every uncountable cardinal κ, graphs of size and chromatic number κ of arbitrarily large odd girth.

For the DM seminar schedule, see:

`http://www.people.vcu.edu/~dcranston/DM-seminar`