VCU Discrete Mathematics Seminar

Wang algebra and spanning trees of receptor dimer models

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1:00-1:50
4145 Harris Hall

In classical electrical network theory, algebraic approaches to enumerating spanning trees of graphs were popular. Let G be an undirected simple graph of size n and, for $1 \leqslant i \leqslant n$, let x_i be an indeterminant associated to edge e_i of G. Let $R = \mathbb{F}_2[x_1, x_2, \ldots, x_n]$ be a polynomial ring over the field with two elements. Define S to be the quotient ring $S = R/I$ where the ideal $I = \langle x_1^2, x_2^2, \ldots, x_n^2 \rangle$ is generated by squares of indeterminants. If $C_1, C_2, \ldots, C_\beta$ are β independent circuits of G, and $h_i = \sum_{j:e_j \in C_i} x_j$ are polynomials in S corresponding to these circuits, then the co-trees (spanning tree complements) of G correspond to the surviving monomials of the product $\Phi_G^*(x_1, \ldots, x_n) = h_1 h_{C_2} \cdots h_{C_\beta}$, which is dual to the Kirchhoff polynomial of G.

My talk will focus on connections between this technique—referred to as a *Wang algebra* by Richard Duffin (1959) and Wai-Kai Chen (1966)—and my research interest in the biophysical theory of ligand-receptor binding. Briefly, receptor oligomers are modeled as identical (but not independent) Markov chains with a state-transition graph that is a reduced Cartesian graph product (Hammack and Smith, 2016). Beginning with a minimal cycle basis construction of such graph products, algebraic enumeration of spanning trees gives insight into receptor dimer models, in particular, thermodynamic constraints on allosteric modulation.