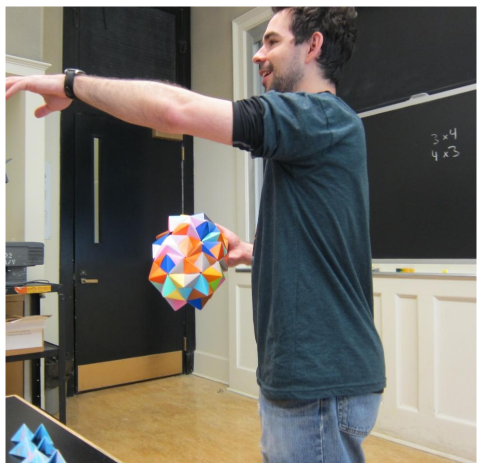
VCU Discrete Mathematics Seminar

Rook theory of the finite general linear group

Prof Joel Brewster Lewis
George Washington University

Friday, Dec. 1
1:00-1:50
4145 Harris Hall



The combinatorial field of rook theory considers the following questions: given a subset B (called a board) of the discrete n-by-n square, how many ways are there to place r rooks on the board so that no two lie in the same row or column? And how many of the full placements of n rooks intersect B in exactly r squares? These numbers, respectively called the rook number and hit number, satisfy a variety of pleasant identities and dualities.

Beginning with work of Garsia and Remmel in the 1980s, combinatorialists have considered q-analogues of these problems, in the following sense: associate to each rook placement P a statistic stat(P), and compute the sum of $q^{\mathrm{stat}(P)}$ over all rook placements, where q is a formal variable. The result is a polynomial in q whose coefficients count rook placements according to stat, and whose value at $q = 1$ is the rook number. Haglund showed that for sufficiently nice boards (the so-called Ferrers diagrams), the Garsia–Remmel q-rook number can also be obtained by counting matrices of a given rank whose support is in the board B over a finite field. In this talk, I'll describe recent work with Morales in which we further investigate this finite field rook theory, including a definition of a q-analogue of the hit numbers and a number of intriguing questions related to positivity of certain formulas.

For the DM seminar schedule, see:

`http://www.people.vcu.edu/~dcranston/DM-seminar.html`