

# VCU Discrete Mathematics Seminar

*Painting squares of graphs with  $\Delta^2 - 1$  colors*

**Prof Dan Cranston**  
**VCU!**

Tuesday, October 7

12:30–1:20

4119 Harris Hall

Brooks' Theorem states that if  $G$  is a connected graph with maximum degree  $\Delta$  at least 3, then  $G$  can be colored with  $\Delta$  colors. This result has been generalized to list-coloring and more general contexts. The *square*  $G^2$  of a graph  $G$  is formed from  $G$  by adding an edge between each pair of vertices at distance two. When  $G$  has maximum degree  $\Delta$ , it is easy to show that  $G^2$  has maximum degree at most  $\Delta^2$ ; so Brooks' Theorem implies that  $G^2$  can be colored with  $\Delta^2$  colors.

Cranston and Kim conjectured that we can improve this upper bound by at least 1. Specifically, they conjectured that  $\chi_\ell(G^2) \leq \Delta^2 - 1$  unless  $G$  is a Moore graph (here  $\chi_\ell$  denotes the list chromatic number). We prove their conjecture and survey some harder conjectures about coloring squares of graphs.

This is joint work with Landon Rabern.



For more information on our fall schedule, see:  
<http://www.people.vcu.edu/~dcranston/DM-seminar/>