Linear Coloring of Sparse Graphs

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Abstract

A linear coloring is a proper coloring such that each pair of color classes induces a union of disjoint paths. We study the linear chromatic number, denoted $\mathrm{lc}(G)$, of sparse graphs. The maximum average degree of a graph G, denoted $mad(G)$, is the maximum of the average degrees of all subgraphs of G. It is clear that any graph G with maximum degree $\Delta(G)$ satisfies $\mathrm{lc}(G) \geq \lceil \Delta(G)/2 \rceil + 1$. In this paper, we prove the following results: (1) if $\mathrm{mad}(G) < 12/5$ and $\Delta(G) \geq 3$, then $\mathrm{lc}(G) = \lceil \Delta(G)/2 \rceil + 1$, and we give an infinite family of examples to show that this result is best possible; (2) if $\mathrm{mad}(G) < 3$ and $\Delta(G) \geq 9$, then $\mathrm{lc}(G) \leq \lceil \Delta(G)/2 \rceil + 2$, and we give an infinite family of examples to show that the bound on $\mathrm{mad}(G)$ cannot be increased in general; (3) if G is planar and has girth at least 5, then $\mathrm{lc}(G) \leq \lceil \Delta(G)/2 \rceil + 4$. In fact, all of our results also hold for linear *list* coloring.